

Primordial gravitational waves from conformal gravity

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Abstract

We investigate the evolution of cosmological perturbations generated during de Sitter inflation in the conformal gravity. Primordial gravitational waves are composed of vector and tensor modes. We obtain the constant vector and tensor power spectra which seems to be correct because the conformal gravity is invariant under conformal transformation like the Maxwell kinetic term.

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1 Introduction

The detection of primordial gravitational waves (GWs) via B-mode polarization of Cosmic Microwave Background Radiation (CMBR) by BICEP2 [1] has shown that the cosmic inflation occurred at a high scale of 10^{16} GeV is the most plausible source of generating primordial GWs. However, more data are required to confirm the above situation. The primordial GWs can be imprinted in the anisotropies and polarization spectrum of CMBR by making the photon redshifts. The B-mode signal observed by BICEP2 might contain contributions from other sources of vector modes and cosmic strings, in addition to tensor modes [2].

The conformal gravity $C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}(=C^2)$ of being invariant under the conformal transformation of $g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu}$ has its own interests in gravity and cosmology. On the gravity side, it gives us a promising combination of $R_{\mu\nu}^2 - R^2/3$ up to the Gauss-Bonnet term which kills massive scalar GWs when it couples to Einstein gravity (known to be the Einstein-Weyl gravity) [3]. Stelle [4] has first introduced the quadratic curvature gravity of $a(R_{\mu\nu}^2 - R^2/3) + bR^2$ to improve the perturbatively renormalizable property of Einstein gravity. In case of $ab \neq 0$, the renormalizability was achieved but the unitarity was violated unless $a = 0$, showing that the renormalizability and unitarity exclude to each other. Although the a -term of providing massive GWs improves the ultraviolet divergence, it induces simultaneously ghost excitations which spoil the unitarity. The price one has to pay for making the theory renormalizable is the loss of unitarity. This issue is not resolved completely until now.

However, the conformal gravity itself is renormalizable. Also, it provides the AdS black hole solution [5] and its thermodynamic properties and holography were discussed extensively in the literature [6, 7, 8]. The authors have investigated the AdS black hole thermodynamics and stability in the Einstein-Weyl gravity and in the limit of the conformal gravity [9].

On the cosmology side of the conformal gravity, it provides surely a massive vector propagation generated during de Sitter inflation in addition to massive tensor ghosts when it couples to Einstein gravity [10, 11, 12]. Recently, the authors have shown that in the limit of $m^2 \rightarrow 0$ (keeping the conformal gravity only), the vector and tensor power spectra disappear. It implies that their power spectra are not gravitationally produced because the vector and tensor perturbations are decoupled from the expanding de Sitter background.

This occurs due to conformal invariance as a transversely massive vector has been shown in the $m^2 \rightarrow 0$ limit of the massive Maxwell theory $(-F^2/4 + m^2 A^2/2)$ [14]. We note here that F^2 is conformally invariant like C^2 under the transformation of $g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu}$ [15]. The conformal gravity implication to cosmological perturbation was first studied in [16] which might indicate that there exists a difference between conformal and Einstein gravities in their perturbed equations around de Sitter background. Even though he has obtained a “degenerate fourth-order equation” for the metric perturbation tensor $h_{\mu\nu}$ from the conformal gravity, any relevant quantity was not found because he did not split $h_{\mu\nu}$ according to the SO(3) decomposition for cosmological perturbations. As far as we know, there is no definite computation of an observable like the power spectrum in the conformal gravity.

In this Letter, we will study the conformal gravity as a higher-order gravity theory to compute the vector and tensor power spectra generated from de Sitter inflation. Considering the conformal invariant of the conformal gravity seriously, we expect to obtain the constant power spectra for vector and tensor perturbations.

2 Conformal gravity

Let us first consider the conformal gravity whose action is given by

$$S_{\text{CG}} = \frac{1}{4\kappa m^2} \int d^4x \sqrt{-g} [C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}], \quad (1)$$

where the Weyl-squared term is given by

$$C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} = 2 \left(R^{\mu\nu} R_{\mu\nu} - \frac{1}{3} R^2 \right) + (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2) \quad (2)$$

with the Weyl tensor

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - \frac{1}{2} (g_{\mu\rho} R_{\nu\sigma} - g_{\mu\sigma} R_{\nu\rho} - g_{\nu\rho} R_{\mu\sigma} + g_{\nu\sigma} R_{\mu\rho}) + \frac{1}{6} R (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}). \quad (3)$$

Here we have $\kappa = 8\pi G = 1/M_{\text{P}}^2$, M_{P} being the reduced Planck mass and a mass-squared m^2 is introduced to make the action dimensionless. Greek indices run from 0 to 3 with conventions $(-+++)$, while Latin indices run from 1 to 3. Further, we note that the Weyl-squared term is invariant under the conformal transformation of $g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu}$.

Its equation takes the form

$$2\nabla^\rho \nabla^\sigma C_{\mu\rho\nu\sigma} + G^{\rho\sigma} C_{\mu\rho\nu\sigma} = 0 \quad (4)$$

with the Einstein tensor $G_{\mu\nu}$. The solution is de Sitter space whose curvature quantities are given by

$$\bar{R}_{\mu\nu\rho\sigma} = H^2(\bar{g}_{\mu\rho}\bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\nu\rho}), \quad \bar{R}_{\mu\nu} = 3H^2\bar{g}_{\mu\nu}, \quad \bar{R} = 12H^2 \quad (5)$$

with $H=\text{constant}$. We choose de Sitter background explicitly by choosing a conformal time η

$$ds_{\text{dS}}^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = a(\eta)^2 \left[-d\eta^2 + \delta_{ij}dx^i dx^j \right], \quad (6)$$

where the conformal scale factor is

$$a(\eta) = -\frac{1}{H\eta} \rightarrow a(t) = e^{Ht}. \quad (7)$$

Here the latter denotes the scale factor with respect to cosmic time t .

We choose the Newtonian gauge of $B = E = 0$ and $\bar{E}_i = 0$ which leads to $10 - 4 = 6$ degrees of freedom (DOF). In this case, the cosmologically perturbed metric can be simplified to be

$$ds^2 = a(\eta)^2 \left[- (1 + 2\Psi)d\eta^2 + 2\Psi_i d\eta dx^i + \left\{ (1 + 2\Phi)\delta_{ij} + h_{ij} \right\} dx^i dx^j \right] \quad (8)$$

with the transverse vector $\partial_i \Psi^i = 0$ and transverse-traceless tensor $\partial_i h^{ij} = h = 0$. We emphasize that choosing the SO(3)-perturbed metric (8) contrasts sharply with the covariant approach to the cosmological conformal gravity [16].

In order to get the cosmological perturbed equations, one is first to obtain the bilinear action and then, varying it to yield the perturbed equations. We expand the conformal gravity action (1) up to quadratic order in the perturbations of Ψ, Φ, Ψ_i , and h_{ij} around the de Sitter background [11]. Then, the bilinear actions for scalar, vector and tensor perturbations can be found as

$$S_{\text{CG}}^{(\text{S})} = \frac{1}{3\kappa m^2} \int d^4x \left[\nabla^2(\Psi - \Phi) \right]^2, \quad (9)$$

$$S_{\text{CG}}^{(\text{V})} = \frac{1}{4\kappa m^2} \int d^4x \left(\partial_i \Psi'_j \partial^i \Psi'^j - \nabla^2 \Psi_i \nabla^2 \Psi^i \right), \quad (10)$$

$$S_{\text{CG}}^{(\text{T})} = \frac{1}{8\kappa m^2} \int d^4x \left(h''_{ij} h''^{ij} - 2\partial_k h'_{ij} \partial^k h'^{ij} + \nabla^2 h_{ij} \nabla^2 h^{ij} \right). \quad (11)$$

Varying the actions (10) and (11) with respect to Ψ^i and h^{ij} leads to the equations of motion for vector and tensor perturbations

$$\square \Psi_i = 0, \quad (12)$$

$$\square^2 h_{ij} = 0, \quad (13)$$

where $\square = d^2/d\eta^2 - \nabla^2$ with ∇^2 the Laplacian operator. It is worth noting that Eqs.(12) and (13) are independent of the expanding de Sitter background in the conformal gravity.

Finally, we would like to mention two scalars Φ and Φ . Two scalar equations are given by $\nabla^2 \Psi = \nabla^2 \Phi = 0$, which implies that they are obviously non-propagating modes in the de Sitter background. This means that the cosmological conformal gravity describes 4 DOF of vector and tensor modes. Hereafter, thus, we will not consider the scalar sector.

3 Primordial power spectra

The power spectrum is given by the two-point correlation function which could be computed when one chooses the vacuum state $|0\rangle$. It is defined by

$$\langle 0 | \mathcal{F}(\eta, \mathbf{x}) \mathcal{F}(\eta, \mathbf{x}') | 0 \rangle = \int d^3 \mathbf{k} \frac{\mathcal{P}_{\mathcal{F}}}{4\pi k^3} e^{-i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}, \quad (14)$$

where \mathcal{F} denotes vector and tensor and $k = |\mathbf{k}|$ is the wave number. In general, fluctuations are created on all length scales with wave number k . Cosmologically relevant fluctuations start their lives inside the Hubble radius which defines the subhorizon: $k \gg aH$ ($z = -k\eta \gg 1$). On the other hand, the comoving Hubble radius $(aH)^{-1}$ shrinks during inflation while the comoving wavenumber k is constant. Therefore, eventually all fluctuations exit the comoving Hubble radius which defines the superhorizon: $k \ll aH$ ($z = -k\eta \ll 1$).

One may compute the two-point function by taking the Bunch-Davies vacuum $|0\rangle$. In the de Sitter inflation, we choose the subhorizon limit of $z \rightarrow \infty$ to define the Bunch-Davies vacuum, while we choose the superhorizon limit of $z \rightarrow 0$ to get a definite form of the power spectrum which stays alive after decaying. For example, fluctuations of scalar and tensor originate on subhorizon scales and they propagate for a long time on superhorizon scales. This can be checked by computing their power spectra which are scale-invariant. Accordingly, it would be interesting to check what happens when one computes the power spectra for vector and tensor perturbations generated from de Sitter inflation in the frame work of conformal gravity.

3.1 Vector power spectrum

Let us consider Eq.(12) for vector perturbation and then, expand Ψ_i in plane waves with the linearly polarized states

$$\Psi_i(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \sum_{s=1,2} p_i^s(\mathbf{k}) \Psi_{\mathbf{k}}^s(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (15)$$

where $p_i^{1/2}$ are linear polarization vectors with $p_i^{1/2} p^{1/2,i} = 1$. Also, $\Psi_{\mathbf{k}}^s$ denote linearly polarized vector modes. Plugging (15) into the equation (12), one finds the equation

$$\left[\frac{d^2}{d\eta^2} + k^2 \right] \Psi_{\mathbf{k}}^s(\eta) = 0. \quad (16)$$

Introducing $z = -k\eta$, Eq.(16) takes a simple form

$$\left[\frac{d^2}{dz^2} + 1 \right] \Psi_{\mathbf{k}}^s(z) = 0 \quad (17)$$

whose positive frequency solution is given by

$$\Psi_{\mathbf{k}}^s(z) \sim e^{iz} \quad (18)$$

up to the normalization.

We are willing to calculate vector power spectrum. For this purpose, we define a commutation relation for the vector. In the bilinear action (10), the conjugate momentum for the field Ψ_j is found to be

$$\pi_{\Psi}^j = -\frac{1}{2\kappa m^2} \nabla^2 \Psi'^j, \quad (19)$$

where one observes an unusual factor ∇^2 which reflects that the vector Ψ_i is not a canonically defined vector, but it is from the cosmological conformal gravity. The canonical quantization is implemented by imposing the commutation relation

$$[\hat{\Psi}_j(\eta, \mathbf{x}), \hat{\pi}_{\Psi}^j(\eta, \mathbf{x}')] = 2i\delta(\mathbf{x} - \mathbf{x}') \quad (20)$$

with $\hbar = 1$.

Now, the operator $\hat{\Psi}_j$ can be expanded in Fourier modes as

$$\hat{\Psi}_j(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \sum_{s=1,2} \left(p_j^s(\mathbf{k}) \hat{a}_{\mathbf{k}}^s \Psi_{\mathbf{k}}^s(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right) \quad (21)$$

and the operator $\hat{\pi}_{\Psi}^j = \frac{k^2}{2\kappa m^2} \hat{\Psi}'^j$ can be easily obtained from (21). Plugging (21) and $\hat{\pi}_{\Psi}^j$ into (20), we find the commutation relation and Wronskian condition as

$$[\hat{a}_{\mathbf{k}}^s, \hat{a}_{\mathbf{k}'}^{s'\dagger}] = \delta^{ss'} \delta^3(\mathbf{k} - \mathbf{k}'), \quad (22)$$

$$\Psi_{\mathbf{k}}^s \left(\frac{k^2}{2\kappa m^2} \right) (\Psi_{\mathbf{k}}^{*s})' - \text{c.c.} = i \rightarrow \Psi_{\mathbf{k}}^s \frac{d\Psi_{\mathbf{k}}^{*s}}{dz} - \text{c.c.} = -\frac{2i\kappa m^2}{k^3}. \quad (23)$$

We choose the positive frequency mode for a Bunch-Davies vacuum $|0\rangle$ normalized by the Wronskian condition

$$\Psi_{\mathbf{k}}^s(z) = \sqrt{\frac{\kappa m^2}{k^3}} e^{iz} \quad (24)$$

as the solution to (17). On the other hand, the vector power spectrum is defined by

$$\langle 0 | \hat{\Psi}_j(\eta, \mathbf{x}) \hat{\Psi}^j(\eta, \mathbf{x}') | 0 \rangle = \int d^3\mathbf{k} \frac{\mathcal{P}_{\Psi}}{4\pi k^3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}, \quad (25)$$

where we take the Bunch-Davies vacuum state $|0\rangle$ by imposing $\hat{a}_{\mathbf{k}}^s|0\rangle = 0$. The vector power spectrum \mathcal{P}_{Ψ} takes the form

$$\mathcal{P}_{\Psi} \equiv \sum_{s=1,2} \frac{k^3}{2\pi^2} |\Psi_{\mathbf{k}}^s|^2. \quad (26)$$

Plugging (24) into (26), we find a constant power spectrum for a vector perturbation

$$\mathcal{P}_{\Psi} = \frac{m^2}{\pi^2 M_{\text{P}}^2}. \quad (27)$$

3.2 Tensor power spectrum

We take Eq.(13) to compute tensor power spectrum. In this case, the metric tensor h_{ij} can be expanded in Fourier modes

$$h_{ij}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \sum_{s=+, \times} p_{ij}^s(\mathbf{k}) h_{\mathbf{k}}^s(\eta) e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (28)$$

where p_{ij}^s linear polarization tensors with $p_{ij}^s p^{s,ij} = 1$. Also, $h_{\mathbf{k}}^s(\eta)$ represent linearly polarized tensor modes. Plugging (28) into (13) leads to the fourth-order differential equation

$$(h_{\mathbf{k}}^s)'''' + 2k^2(h_{\mathbf{k}}^s)'' + k^4 h_{\mathbf{k}}^s = 0, \quad (29)$$

which is further rewritten as a factorized form

$$\left[\frac{d^2}{d\eta^2} + k^2 \right]^2 h_{\mathbf{k}}^s(\eta) = 0. \quad (30)$$

Introducing $z = -k\eta$, Eq.(30) takes the form of a degenerate fourth-order equation

$$\left[\frac{d^2}{dz^2} + 1\right]^2 h_{\mathbf{k}}^s(z) = 0. \quad (31)$$

This is the same equation for a degenerate Pais-Uhlenbeck (PU) oscillator and its solution is given by

$$h_{\mathbf{k}}^s(z) = \frac{N}{2k^2} \left[(a_2^s + a_1^s z) e^{iz} + c.c. \right] \quad (32)$$

with N the normalization constant. After quantization, a_2^s and a_1^s are promoted to operators $\hat{a}_2^s(\mathbf{k})$ and $\hat{a}_1^s(\mathbf{k})$. The presence of z in (\dots) reflects clearly that $h_{\mathbf{k}}^s(z)$ is a solution to the degenerate equation (31). However, it is difficult to quantize h_{ij} in the subhorizon region directly because it satisfies the degenerate fourth-order equation (13). In order to quantize h_{ij} , we have to consider (13) as a final equation obtained by making use of an auxiliary tensor β_{ij} .

For this purpose, one rewrites the fourth-order action (11) as a second-order action

$$S_{\text{AC}}^{(\text{T})} = -\frac{1}{4\kappa m^2} \int d^4x \left(\eta^{\mu\nu} \partial_\mu \beta_{ij} \partial_\nu h^{ij} + \frac{1}{2} \beta_{ij} \beta^{ij} \right). \quad (33)$$

Their equations are given by

$$\square h_{ij} = \beta_{ij}, \quad \square \beta_{ij} = 0, \quad (34)$$

which are combined to give the fourth-order tensor equation (13). Explicitly, acting \square on the first equation leads to (13) when one uses the second one. Actually, this is an extension of the singleton action describing a dipole ghost pair as the fourth-order scalar theory [17, 18, 19, 20]. This is related to not a non-degenerate PU oscillator and its quantization, but a degenerate PU and quantization [21, 22]. The canonical conjugate momenta are given by

$$\pi_h^{ij} = \frac{1}{4\kappa m^2} \beta^{ij}, \quad \pi_\beta^{ij} = \frac{1}{4\kappa m^2} h^{ij}. \quad (35)$$

After expanding \hat{h}_{ij} and $\hat{\beta}_{ij}$ in their Fourier modes, their amplitudes at each mode are given by

$$\hat{\beta}_{\mathbf{k}}^s(z) = iN \left(\hat{a}_1^s(\mathbf{k}) e^{iz} - \hat{a}_1^{s\dagger}(\mathbf{k}) e^{-iz} \right), \quad (36)$$

$$\hat{h}_{\mathbf{k}}^s(z) = \frac{N}{2k^2} \left[(\hat{a}_2^s(\mathbf{k}) + \hat{a}_1^s(\mathbf{k}) z) e^{iz} + \text{h.c.} \right]. \quad (37)$$

Now, the canonical quantization is accomplished by imposing equal-time commutation relations:

$$[\hat{h}_{ij}(\eta, \mathbf{x}), \hat{\pi}_h^{ij}(\eta, \mathbf{x}')] = 2i\delta^3(\mathbf{x} - \mathbf{x}'), \quad [\hat{\beta}_{ij}(\eta, \mathbf{x}), \hat{\pi}_\beta^{ij}(\eta, \mathbf{x}')] = 2i\delta^3(\mathbf{x} - \mathbf{x}'), \quad (38)$$

where the factor 2 is coming from the fact that h_{ij} and β_{ij} represent 2 DOF, respectively. Taking (36) and (37) into account, the two operators $\hat{\beta}_{ij}$ and \hat{h}_{ij} are given by

$$\hat{\beta}_{ij}(z, \mathbf{x}) = \frac{N}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \left[\sum_{s=+, \times} \left(ip_{ij}^s(\mathbf{k}) \hat{a}_1^s(\mathbf{k}) e^{iz} e^{i\mathbf{k}\cdot\mathbf{x}} \right) + \text{h.c.} \right], \quad (39)$$

$$\hat{h}_{ij}(z, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3\mathbf{k} \frac{N}{2k^2} \left[\sum_{s=+, \times} \left\{ p_{ij}^s(\mathbf{k}) \left(\hat{a}_2^s(\mathbf{k}) + \hat{a}_1^s(\mathbf{k}) z \right) e^{iz} e^{i\mathbf{k}\cdot\mathbf{x}} \right\} + \text{h.c.} \right]. \quad (40)$$

Plugging (39) and (40) into (38) determines the normalization constant $N = \sqrt{2\kappa m^2}$ and commutation relations between $\hat{a}_i^s(\mathbf{k})$ and $\hat{a}_j^{s'\dagger}(\mathbf{k}')$ as

$$[\hat{a}_i^s(\mathbf{k}), \hat{a}_j^{s'\dagger}(\mathbf{k}')] = 2k\delta^{ss'} \begin{pmatrix} 0 & -i \\ i & 1 \end{pmatrix} \delta^3(\mathbf{k} - \mathbf{k}'). \quad (41)$$

Here the commutation relation of $[\hat{a}_2^s(\mathbf{k}), \hat{a}_2^{s'\dagger}(\mathbf{k}')] is determined by the condition of$

$$[\hat{h}_{ij}(\eta, \mathbf{x}), \hat{\pi}_{\beta}^{ij}(\eta, \mathbf{x}')] = 0. \quad (42)$$

We are ready to compute the power spectrum of the gravitational waves defined by

$$\langle 0 | \hat{h}_{ij}(\eta, \mathbf{x}) \hat{h}^{ij}(\eta, \mathbf{x}') | 0 \rangle = \int d^3k \frac{\mathcal{P}_h}{4\pi k^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}. \quad (43)$$

Here we choose the Bunch-Davies vacuum $|0\rangle$ by imposing $\hat{a}_i^s(\mathbf{k})|0\rangle = 0$. The tensor power spectrum \mathcal{P}_h in (43) denotes $\mathcal{P}_h \equiv \sum_{s=+, \times} \mathcal{P}_h^s$ where \mathcal{P}_h^s is given as

$$\mathcal{P}_h^s = \frac{k^3}{2\pi^2} |h_{\mathbf{k}}^s|^2 = \frac{m^2}{2\pi^2 M_{\text{P}}^2}. \quad (44)$$

Finally, we obtain the tensor power spectrum

$$\mathcal{P}_h = \frac{m^2}{\pi^2 M_{\text{P}}^2} \quad (45)$$

which corresponds to a constant power spectrum. This is the same form as for the vector power spectrum (27).

On the other hand, the power spectrum of auxiliary tensor β_{ij} is defined by

$$\langle 0 | \hat{\beta}_{ij}(\eta, \mathbf{x}) \hat{\beta}^{ij}(\eta, \mathbf{x}') | 0 \rangle = \int d^3k \frac{\mathcal{P}_{\beta}}{4\pi k^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}. \quad (46)$$

Here we obtain the zero power spectrum as

$$\mathcal{P}_\beta = 0 \quad (47)$$

when one used the commutation relation $[\hat{a}_1^s(\mathbf{k}), \hat{a}_1^{s'\dagger}(\mathbf{k}')] = 0$. This is clear because β_{ij} is an auxiliary tensor to lower the fourth-order action to the second-order action. However, it is not understood why \hat{h}_{ij} could be expanded by $\hat{a}_2^s(\mathbf{k})$ and $\hat{a}_1^s(\mathbf{k})$ without introducing β_{ij} , because β_{ij} was expanded by $\hat{a}_1^s(\mathbf{k})$ solely.

4 Discussions

We have found the constant vector and tensor power spectra generated during de Sitter inflation from conformal gravity. These constant power spectra could be understood because the conformal gravity is invariant under conformal (Weyl) transformation. This means that their power spectra are constant with respect to $z = -k\eta$ since vector and tensor perturbations are decoupled from the expanding de Sitter inflation. In other words, this is so because the bilinear actions (10) and (11) are independent of the conformal scale factor $a(\eta)$ as a result of conformal invariance. On the contrary of Ref.[16], it is less interesting for the conformal gravity to further investigate its cosmological implications.

Hence, our analysis implies that the Einstein-Weyl gravity is more promising than the conformal gravity to obtain the physical tensor power spectrum because the Einstein-Hilbert term provides the coupling of scale factor a like as $a^2(h'_{ij}h'^{ij} - \partial_l h_{ij}\partial^l h^{ij})$. Also, the singleton Lagrangian of $\mathcal{L}_s = -\sqrt{-g}(\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_2 + \frac{1}{2}\phi_1^2)$ is quite interesting because it provides two scalar equations ($\square + 2aH\partial_\eta\phi_2 = \phi_1$ and $(\square + 2aH\partial_\eta)\phi_1 = 0$ which are combined to yield the degenerate fourth-order equation $(\square + 2aH\partial_\eta)^2\phi_2 = 0$. Here we observe the presence of the scale factor a in the perturbed equation of the singleton.

Consequently, the conformal invariance of the Lagrangian like $\sqrt{-g}C^2$ or $\sqrt{-g}F^2$ has no responsibility for generating the observed fluctuations during inflation.

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